Impact Phenomena

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A statistical interpretation of experimental results derived from a relatively small number of test-shots strongly indicated that the motion of penetrating bodies might be governed by the combined result of at least three different kinds of laws of resistance acting simultaneously upon the moving body. Considering the fact that during the penetration the relative velocity between the penetrating body and the target changes from a maximum speed to zero (in the case of a quasi-infinite target), the three laws of resistance have been chosen as functions of the zero, first, and second power of the instantaneous velocity. As a result, a differential equation has been adopted as a working hypothesis. Finally, suitably chosen integrations lead to equations that describe penetration depth and penetration time as a function of the instantaneous velocity. A comparison between theoretically predicted results and experimental results is shown.

MANY attempts have been made to describe impact phenomena quantitatively. To deepen our understanding and to support new theoretical approaches, numerous experiments have been conducted. A comprehensive collection of experimental data, as well as a critical review on theoretical approaches, has been published by Herrmann and Jones.[1]

The problem of describing impact phenomena, in particular those resulting from hypervelocity impacts, is a complex one and has not yet been solved rigorously. Moreover, there is a good reason to presume that there never will be an ideal solution to this problem, but rather approximations well suited for certain groups of such phenomena.

In 1956 and 1957 a theory concerning impact phenomena was developed with the initial intent of being an aid for the interpretation of laboratory experiments conducted in connection with a research project on the determination of the mass and spatial distribution of micrometeorites in the vicinity of the earth. This theory deals primarily with the determination of the penetration depth and the penetration time, assuming normal impact on quasi-infinite targets. No other assumptions whatever have been made. The applicability of this theory therefore is restricted neither to certain kinds of materials nor to certain ranges of impact velocities.

Two publications on that topic have been made previously by Bohn and Fuchs in 1958[2] and in 1960.[3] Another description of said theory can be found in the addendum of the Herrmann-Jones report,[1] Chap. 3.1.6, p. 20. In the latter report, unfortunate circumstances were responsible for a misinterpretation of the basic concept.

In this paper, an attempt will be made to present the theory again in a short form in order to clarify a few points that might have caused some confusion with regard to the basic concept as well as to the practical application of that theory.

According to the fundamental concept adopted, the resistive forces $R(v)$ between the mass of the projectile, m, on the one hand and the target material on the other hand are basically the combined result of three components: $f_1(v_0)$, $f_2(v^1)$, and $f_3(v^2)$, respectively.

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These three terms are functions of the instantaneous velocity v so that, in accordance with Newton's second law of motion, the instantaneous forces of resistance $R(v)$ will be

$$R(v) = f_1(v^0) + f_2(v^1) + f_3(v^2) = -m(dv/dt) \qquad (1)$$

Equation (1) means implicitly that, during the first phase of penetration (the projectile having high velocity), the motion is governed mainly by resistive forces proportional to the square of the velocity. In the following phase, at moderate velocities, the deceleration will be proportional mainly to the first power of the velocity, and in the final phase the deceleration is caused mainly by static forces.

The first term of Eq. (1), namely, $f_1(v^0)$, has been determined as the product

$$f_1(v^0) = a \cdot H \qquad (2)$$

where a means the cross-sectional area of the projectile measured in a plane perpendicular to the path of flight, and H stands for a stress factor not yet defined.

As the results of a series of crucial test-shots (BB-projectiles fired from a gas carbine at speeds between 90 and 200 m-sec^{-1}) indicated, the stress factor H is not identical with the Brinell hardness B. However, it has been found that, by substituting the Brinell hardness B for the unknown quantity H, fairly good approximations might be obtained in most of the cases. It seems advisable to say a few more words about these quantities B and H, respectively.

It is well known that the so-called Brinell hardness B is defined by the formula

$$B = \frac{2F}{\pi D[D - (D^2 - d^2)^{1/2}]} \qquad (3)$$

In this expression, F is the force exerted on a sphere, D is the diameter of the sphere, and d signifies the diameter of identation which remains on the surface of the material examined as the result of the inelastic deformation caused by the sphere.

There is no strict relationship between certain stress properties of the material examined and the Brinell hardness B, simply because the duration of the pressures as well as the diameter D are affecting the quantity B. For engineering purposes, certain rules have been established in order to determine what forces, durations, and diameters of the spheres have to be chosen for particular types of materials. Without reference to such rules, the Brinell hardness B as derived from Eq. (3) does not make much sense.

In order to establish a firm relationship between the "static" forces given by $f_1(v^0)$ and the stress factor H, new rules have to be set up. No attempt has been made as yet to determine such rules or to devise a new procedure for the determination of H. In case very reliable predictions as to penetration depths and/or penetration times at high velocity ranges are a must, it is advisable to determine H quantitatively by an experiment, carried out at an arbitrarily chosen low velocity or vice versa. If reliable values for the quantity H are not available and if the determination of H by an experiment is not feasible, the Brinell hardness B might replace H as an approximation.

The third term $f_3(v^2)$ has been defined in analogy to aerodynamics.

$$f_3(v^2) = C \cdot a \cdot (\gamma_t/2 \cdot g)v^2 \qquad (4)$$

in which C is a shape factor (also in analogy to aerodynamics), γ_t density of the target material, g acceleration due to gravity, and v the instantaneous velocity of the center of gravity of the projectile with respect to the target as a whole.

It is expected that in most of the cases the shape factor C has to be determined by experiment rather than by theoretical speculation. Nevertheless, an attempt has been made to determine approximated C values for certain shapes by analytical means. For that purpose it was assumed that the shape factor is a function of the angle at which a section of the surface of the projectile is inclined toward the direction of flight.

Table 1 shows estimated values for the shape factor C for three basic types of shapes. The C values smaller than unity should be employed only if the stress factor of the projectile H_{proj} or the Brinell hardness B_{proj}—whichever quantity is available—is larger than the corresponding values of the target material.

In analogy to Stoke's law, finally, the middle term of Eq. (1), namely, $f_2(v^1)$, describes that component of the resistive force which governs the motion of the projectile mainly at moderate speeds.

At this stage, a somewhat arbitrary step has been taken in order to achieve a most simple expression for the total resistance at a given instant. According to Eq. (1), one can now write

$$R(v) = a \cdot H + f_2(v^1) + (\gamma_t/2 \cdot g)C \cdot a \cdot v^2 \qquad (5)$$

Furthermore, substituting the symbol α for $f_1(v^0)$ and $\beta \cdot v^2$ for $f_3(v^2)$, Eq. (5) can be written as

$$R(v) = \alpha + f_2(v^1) + \beta v^2 \qquad (6)$$

Theroetically, the symbol $f_2(v^1)$ stands for an expression that is rather difficult to handle. Fortunately, it has been found that the product $(2\alpha^{1/2}\beta^{1/2}v^1)$ can serve as a good approximation for the middle term $f_2(v^1)$.

The arbitrary step, mentioned in the foregoing, consists of the decision to adopt the substitution

$$f_2(v^1) = 2\alpha^{1/2}\beta^{1/2}v^1 \qquad (7)$$

which leads to the simple expression for the instantaneous resistive force

$$R(v) = (\alpha^{1/2} + \beta^{1/2}v)^2 = -m(dv/dt) = -m(d^2x/dt^2) \qquad (8)$$

where m is the mass of the projectile and dv/dt is the instantaneous acceleration of the projectile. Equation (8) represents the so-called "simplified version" of the theory in question.

Determination of Penetration Depth x

By multiplying Eq. (8) with the differential dx, one obtains

$$(\alpha^{1/2} + \beta^{1/2}v)^2 dx = -mvdv \qquad (9)$$

hence one gets

$$dx = -m[v\, dv/(\alpha^{1/2} + \beta^{1/2}v)^2] \qquad (10)$$

Table 1 Values for the shape factor C

Shape	Approximated value
Cube or inscribed cylinder	1.0
Sphere	$\frac{2}{3}$
Pointed projectile, like armour-breaking artillery shell	$\frac{1}{3}$

The integration of Eq. (10) leads to

$$x = -m \int \frac{vdv}{(\alpha^{1/2} + \beta^{1/2}v)^2} = - m \left\{\frac{1}{\beta}\left[\ln(\alpha^{1/2} + \beta^{1/2}v) + \frac{\alpha^{1/2}}{(\alpha^{1/2} + \beta^{1/2}v)}\right]\right\} + C_1 \qquad (11)$$

Since $x = 0$ if $v = v_0$ (v_0 being the initial velocity), one finds

$$C_1 = \frac{m}{\beta}\left\{\ln(\alpha^{1/2} + \beta^{1/2}v_0) + \frac{\alpha^{1/2}}{(\alpha^{1/2} + \beta^{1/2}v_0)}\right\} \qquad (12)$$

The instantaneous penetration depth x therefore is

$$x(v) = \frac{m}{\beta}\left\{\ln\frac{\alpha^{1/2} + \beta^{1/2}v_0}{\alpha^{1/2} + \beta^{1/2}v} - \alpha^{1/2}\beta^{1/2}\frac{v_0 - v}{(\alpha^{1/2} + \beta^{1/2}v_0)(\alpha^{1/2} + \beta^{1/2}v)}\right\} \qquad (13)$$

Assuming a quasi-infinite thickness of the target, the final velocity of the projectile will be equal to zero. For $v = 0$, the maximum penetration depth x^* will be

$$x^*(v_0) = \frac{m}{\beta}\left\{\ln\left(1 + \frac{\beta^{1/2}}{\alpha^{1/2}}v_0\right) - \frac{\beta^{1/2}v_0}{(\alpha^{1/2} + \beta^{1/2}v_0)}\right\} \qquad (14)$$

Determination of Penetration Time T

Here the basic equation of the simplified version of the theory, namely Eq. (8), will be multiplied by the differential dt. Hence one obtains

$$dt = -m[dv/(\alpha^{1/2} + \beta^{1/2}v)^2] \qquad (15)$$

By integration of Eq. (15), one obtains for the time of penetration

$$T(v) = -m\left\{-\frac{1}{\beta^{1/2}}\frac{1}{(\alpha^{1/2} + \beta^{1/2}v)}\right\} + C_2 \qquad (16)$$

Since at the moment of impact $T = 0$ and $v = v_0$, the constant C_2 becomes

$$C_2 = m\left\{-\frac{1}{\beta^{1/2}}\frac{1}{(\alpha^{1/2} + \beta^{1/2}v_0)}\right\} \qquad (17)$$

The elapsed time T between the instant of impact and the instant at which the projectile has attained a velocity v will be

$$T(v) = m\left\{\frac{v_0 - v}{(\alpha^{1/2} + \beta^{1/2}v_0)(\alpha^{1/2} + \beta^{1/2}v)}\right\} \qquad (18)$$

For a quasi-infinite target, the maximum penetration time T^* will be obtained by making v equal to zero:

$$T^*(v_0) = \frac{m}{\alpha^{1/2}}\frac{v_0}{(\alpha^{1/2} + \beta^{1/2}v_0)} \qquad (19)$$

In Figs. 1 and 2, four sets of experimental results are shown on graphs accompanied by the corresponding theoretical functions computed according to Eq. (14). These experimental results, originally obtained by various laboratories, have been selected in a random manner from the addendum to the report of Herrmann and Jones.[1] The reference and page numbers shown on the figures refer to that

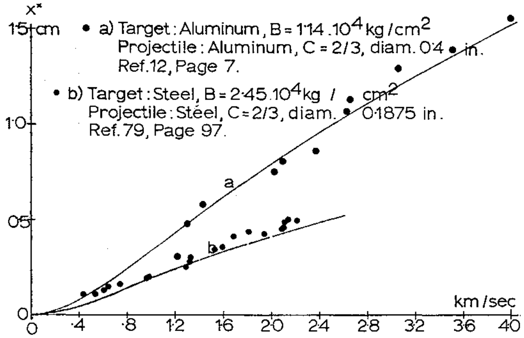

Fig. 1 Two comparisons between theory and experimental results. Target and projectile are of same material.

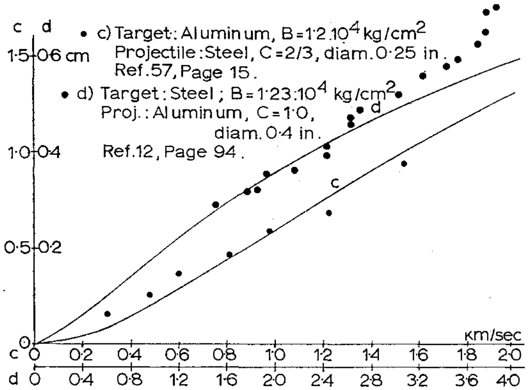

Fig. 2 Two comparisons between theory and experimental results. Target and projectile are of different materials.

addendum. The theoretical functions as shown in these graphs are not entirely identical with those shown in the Herrmann-Jones report. Moreover, the theoretical functions had to be re-evaluated for several reasons. For instance, the Herrmann-Jones report shows, in many of the graphs, data obtained with spherical projectiles and with cylindrical projectiles plotted on the same diagram while showing the theoretical function for only one of the shapes. Furthermore, a few experimental points (read-out of penetration depth) had to be omitted because they were obviously erroneous.

In recomputing the theoretical functions, no attempt has been made to achieve a better correspondence between the theoretical functions and the experimental data. Throughout, the shape factor C has been chosen according to the approximate values shown in Table 1, whereas the stress factor H was taken to be equal to the Brinell hardness B as reported by the various laboratories whose work was analyzed in the addendum to the Herrmann-Jones report.[1]

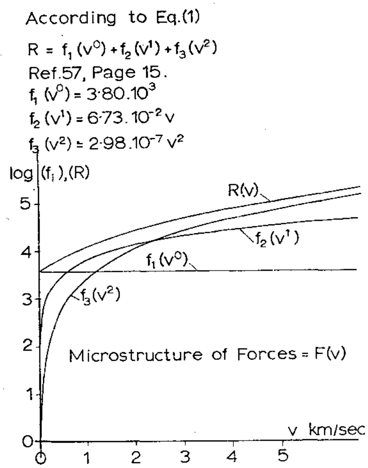

Fig. 3 Three components of the resistive force *R* and their combined results.

The three components of the resistive force R and their combined results have been determined numerically for a particular case and are shown on Fig. 3.

Conclusion

Although approximate values for the stress factor H and the shape factor C have been employed for the determination of the theoretical functions, the simplified version of the theory describes fairly well the experimental results.

References

[1] Herrmann, W. and Jones, A. H., "Survey of hypervelocity impact information," Aeroelastic and Structures Research Lab. Rept. 99-1, including Addendum (September 1961); also "Correlation of hypervelocity impact data," paper presented at Fifth Hypervelocity Impact Symposium (1961).

[2] Bohn, J. L. and Fuchs, O. P., "High velocity impact studies directed toward the determination of the spatial density, mass and velocity of micrometeorites at high altitudes," Sci. Rept. 1, Air Force Cambridge Research Center TN-58-243, Armed Services Tech. Info. Agency, Doc. AD 152478 (January 1958).

[3] Bohn, J. L., Fuchs, O. P., Hewitt, E., and Sherwood, E. J., "Research directed towards the study of transducing meteoric impacts," Final Rept., Air Force Cambridge Research Labs. TR-60-436 (December 1960).